

# Narrow Money, Broad Money, and the Transmission of Monetary Policy

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## Abstract

The paper investigates the role of broad liquidity—the supply and demand for bank deposits—in the transmission of monetary policy. The model integrates deposit demand, loan production, asset pricing, and arbitrage between banking and asset markets. Broad liquidity conditions must be taken into account in the pursuit of interest rate policy for two reasons: (1) they influence the link between the interbank rate and market rates through their effect on the external finance premium, and (2) they affect the behavior of market rates that the central bank must target in order to maintain price stability. The paper shows how the production and use of broad liquidity influences the “*neutral*” interbank rate consistent with balanced growth and stable inflation. It shows how and why interbank rate policy *actions* must be modified in light of broad liquidity considerations to stabilize inflation in response to shocks.

## 1 Introduction

Monetary policy is commonly examined in the context of models with a simplified monetary transmission mechanism without any role for money itself.<sup>1</sup> There is nothing necessarily wrong in this. Interest rate rules for monetary policy have been shown to deliver coherent outcomes for the price level and real variables, even in models that ignore the demand and supply for monetary aggregates completely.<sup>2</sup> Moreover, central

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<sup>1</sup>For example, see Brayton et al. (1997).

<sup>2</sup>See, for example, Kerr and King (1996), McCallum (2001), and Woodford (2003).

banks ordinarily focus on interest rates in the transmission of monetary policy. Furthermore, the demand for narrow money (currency and bank reserves) is automatically accommodated at the current interest rate target so that interest rate policy need not be modified to take account of narrow monetary conditions. This paper shows, however, that *interest rate policy must be modified to take account of broad monetary conditions* involving the supply and demand for bank deposits. To put it another way: a model of monetary policy that ignores broad liquidity considerations is incomplete and misleading as a guide to interest rate policy.<sup>3</sup>

In order to investigate the role of broad liquidity provision in the transmission of monetary policy, the paper models the supply and demand for bank deposits in some detail. The model integrates broad money demand, loan production, asset pricing, and arbitrage between banking and asset markets. The aim of the paper is to build a model rich enough but simple enough to understand the various channels by which shocks impact conditions of supply and demand for broad money and influence the way in which a central bank must conduct interest rate policy.

Households hold bank deposits in the model to self-insure against liquidity risk given that consumption must be chosen before income is realized and households can only rebalance their portfolios at the beginning of each period. Households borrow from banks to fund beginning-of-period deposits. Loans are produced with two inputs: management effort and loan collateral. Because assets provide collateral services in loan production, their total return is the sum of an implicit broad liquidity services component and the usual explicit pecuniary component.

The heart of the model is the simultaneous determination of the consumption price of physical capital  $q$  and management effort  $m$  in loan production. These key variables must satisfy two equilibrium conditions: (1) a broad liquidity condition which requires loan production to equal deposit demand in excess of bank reserve demand, and (2) a capital asset pricing condition that requires the implicit broad liquidity services yield on capital plus the risk-adjusted expected pecuniary yield to equal the required total yield on assets that is consistent with macroeconomic equilibrium. There is an external finance premium, dependent on  $q$  and  $m$ , that links the bank loan rate to the cost of loanable funds in general, and to the interbank interest rate in particular. A no arbitrage condition requires the loan rate to equal the total risk-adjusted nominal return on assets consistent with macroeconomic equilibrium. The model is employed to show how the broad liquidity considerations outlined above must be taken into account when monetary policy geared to maintaining price stability is implemented with an interbank rate policy instrument.

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<sup>3</sup>Ireland (2001), Leeper and Roush (2003), and Nelson (2002) suggest some theoretical reasons and provide some empirical evidence for why money is not redundant in the transmission of monetary policy given interest rates. Goodfriend (1999) contains a related discussion.

The paper builds on the following work in macroeconomics. First, the paper adapts the theoretical framework that Poole (1968) used to study the management of bank reserves under uncertainty to model the demand for deposits by households. Second, the paper builds on the work of Bernanke and Gertler (1995) to model the external finance premium and the productive role of collateral in loan production. Third, the paper builds on Keynes (1936) and Friedman (1969) to identify the implicit broad liquidity services yield of bonds and physical capital with their productive role as collateral in loan production.<sup>4</sup> Fourth, the paper builds on Lucas (1978) to determine asset prices and returns, interest rate spreads, and the division of returns between the implicit broad liquidity services yield and the explicit pecuniary yield.<sup>5</sup> Fifth, asset markets play a role in the model reminiscent of Meltzer’s (1995) monetarist view of monetary policy transmission.

The structure of the paper is as follows. Broad money demand is derived in Section 2. The loan production technology is presented, the external finance premium is introduced, and equilibrium conditions for the banking sector are characterized in Section 3. The first order conditions for the household utility maximization problem are derived in Section 4. Market clearing conditions and preliminary solutions for the external finance premium are presented in Section 5. The demand for currency is derived and price level determination is discussed in Section 6. Preliminary solutions for interest rates, interest rate spreads, implicit broad liquidity service yields, and explicit pecuniary yields are discussed in Section 7. The price of capital  $q$  and loan production effort  $m$  are determined in Section 8 in balanced growth with aggregate perfect foresight. The model is then used to illustrate the consequences of the production and use of broad liquidity for the level of the “*neutral*” interbank interest rate target. Section 9 shows how and why interest rate policy *actions* must be modified to take account of broad liquidity considerations for a variety of shocks to the macroeconomy. A brief summary concludes the paper.

## 2 The Demand for Broad Money

Households acquire bank deposits in the model to self-insure against broad liquidity risk. Households are exposed to liquidity shocks because consumption in a period must be chosen before household income is realized, and consumption must be paid for with income realized during the period, with bank deposits held at the beginning of the period, or by overdrafting a deposit account at the end of a period. Assume that households are confident of the central bank’s commitment to price stability so that the

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<sup>4</sup>See Keynes (1936), Chapter 17 and Friedman (1969), Chapter 1.

<sup>5</sup>See Bansal and Coleman (1996) for an asset pricing model that distinguishes between explicit pecuniary and implicit service yields to study the equity premium.

price level  $P$  is known with certainty at the start of a period. Then, a household can meet its liquidity needs at the end of a period if  $\frac{D}{P} + y - c > 0$ , where  $D$  is the nominal stock of deposits held by a household at the beginning of the period,  $y$  is real household income earned during the period, and  $c$  is real household consumption spending during the period. On the other hand, if  $\frac{D}{P} + y - c < 0$ , then a household must satisfy its end-of-period need for liquidity by overdrafting its bank account.

The net opportunity cost (in nominal terms) of being stuck with a dollar of excess deposits at the end of a period is  $R^T - R^D$ , where  $R^T$  is the total nominal yield foregone by holding deposits instead of a non-monetary asset, and  $R^D$  is the nominal interest rate paid on deposits carried into the following period. The net opportunity cost per dollar of a deficiency of deposits at the end of a period and having to overdraft is  $R^{OD} - R^T$ , where  $R^{OD}$  is the nominal interest cost of overdrafts.

Throughout the paper we assume that aggregate income and consumption are known contemporaneously by households at the beginning of each period and that liquidity risk facing households stems entirely from unforecastable idiosyncratic income shocks. We use the liquidity risk facing the average household  $y = c(1 + \varepsilon)$ , where  $\varepsilon$  is a zero mean random variable, to express the random net liquidity inflow scaled by consumption as  $\frac{y-c}{c} = \varepsilon$ . Furthermore, we assume that a household chooses beginning-of-period deposits  $D^*$  to minimize the expected cost due to uncertainty about deposit gains and losses during a period.<sup>6</sup> Following Poole (1968) the solution  $D^*$  to this deposit demand problem can be characterized implicitly by

$$\Pr\left[\frac{y-c}{c} < -\frac{D^*}{cP}; \sigma_{\frac{y-c}{c}}\right] = \frac{R^T - R^D}{R^{OD} - R^D} \quad (1)$$

The solution says that a household chooses its beginning-of-period real stock of deposits  $\frac{D^*}{P}$  scaled by household consumption  $c$  so that the probability of overdrafting to meet its current consumption expenses equals the expression on the right hand side of the condition. Note the following features of the demand for deposits implied by (1). First, condition (1) determines real deposit demand. Second, real deposit demand depends negatively on  $R^T$  and positively on  $R^{OD}$ . Third,  $D^* > 0 \rightarrow \frac{R^T - R^D}{R^{OD} - R^D} < \frac{1}{2} \rightarrow \frac{R^{OD}}{R^T} > 1$ . Fourth, using the  $R^{OD} > R^T$  condition, real deposit demand varies directly with  $R^D$ .

For what follows it is useful to express the real demand for deposits in terms of desired velocity as a function of the right hand side of (1)

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<sup>6</sup>The assumption of risk neutrality here simplifies the characterization of the demand for deposits with little effect on subsequent analysis even though households are assumed to have log utility in Section 4.

This model of deposit demand has much in common with Bewley's incomplete markets model of money demand. See Ljungqvist and Sargent (2000), Chapter 14 and references therein for a discussion of incomplete market models.

$$\frac{D^*}{P} = c/V[\frac{R^T - R^D}{R^{OD} - R^D}] \quad (2)$$

where  $V[\frac{R^T - R^D}{R^{OD} - R^D}]$  is the desired consumption velocity of deposits and  $V'[\frac{R^T - R^D}{R^{OD} - R^D}] > 0$ . In this paper we assume that real deposit demand adjusts costlessly and immediately at the beginning of each period to consumption and desired velocity.<sup>7</sup>

### 3 The Banking Sector

Households borrow from banks in order to fund deposits. Loans are produced with effort to manage and monitor the extension of credit. Effort is more productive in making loans the greater the borrower's collateral. Collateral is a valuable input in loan production because it enables a bank to enforce the repayment of loans with less monitoring and management effort.<sup>8</sup> The loan production technology is assumed to be Cobb-Douglas in weighted collateral  $\frac{B}{P} + kqK$  and monitoring and managing effort  $m$

$$\frac{L}{P} = F(\frac{B}{P} + kqK)^\alpha (\gamma dm)^{1-\alpha} \quad (3)$$

where  $L$  is the dollar volume of loans extended to a household,  $P$  is the price level,  $F$  is a loan productivity coefficient,  $B$  is the nominal stock of government bonds held by a household,  $K$  is units of productive physical capital held by a household,  $m$  is effort expended on monitoring and managing the household's loans,  $q$  is the consumption price of capital,  $k < 1$  is a factor weighting the productivity of physical capital relative to government bonds as loan collateral,  $d$  is an index of economy-wide productivity that will be identified with the dividend on capital below, and  $\gamma$  is a factor governing the productivity of monitoring effort in producing loans. Deposits do not provide collateral services in loan production because their availability to pay for consumption means that they cannot be pledged as collateral. Currency, to be introduced below, provides no collateral services for the same reason.

A household's external finance premium  $EFP$  depends on the amount of borrowing it chooses to do relative to its collateral. According to loan production technology (3),

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<sup>7</sup>Adjustment costs of rebalancing portfolios between broad money and other assets are important for understanding the dynamics of velocity and broad money demand in practice. Christiano and Eichenbaum's (1992) work on the liquidity effect highlights the important role played by portfolio adjustment costs.

<sup>8</sup>Banks expend effort and require borrowers to post collateral but in equilibrium there is no default. This modelling choice is based, in part, on the progress that has been made in understanding the implications of credit market imperfections in limited commitment environments such as Kocherlakota (1996) where there is no equilibrium default.

the marginal loan in nominal terms taken out by a household incurs an external finance premium of

$$EFP = \frac{W(\gamma dm)^\alpha}{(1 - \alpha)PF(\frac{B}{P} + kqK)^\alpha \gamma d} \quad (4)$$

where  $W$  is the nominal wage and the  $EFP$  is the marginal cost in hours of monitoring and managing effort multiplied by the nominal wage. Equivalently, the  $EFP$  is the nominal marginal cost of managing and monitoring a loan for one period. Hence, the  $EFP$  may be thought of as one component of the total per period nominal interest cost of a nominal loan. The other component is the cost of a dollar of loanable funds  $\frac{R^D}{1 - rr}$  which is the nominal interest rate paid on deposits  $R^D$  divided by  $1 - rr$ , where  $rr$  is the ratio of non-interest-paying reserves to deposits held by banks. Profit maximization and competition among banks ensures that the nominal interest rate on loans equals the nominal marginal cost of loan production:  $R^L = EFP + \frac{R^D}{1 - rr}$ .

We finish characterizing equilibrium conditions in banking markets as follows. First, a no arbitrage condition must be satisfied between the loan market and the asset market:  $R^L = R^T$ , where  $R^T$  is the total nominal yield on assets mentioned above. Second, the no arbitrage condition together with the determinants of the loan rate simplify the determination of the demand for deposits as a function of velocity in (2) as follows. Write the interest rate on overdrafts in terms of a fixed overdraft premium  $OP$ :  $R^{OD} = (OP)EFP + \frac{R^D}{1 - rr}$ , where  $D^* > 0 \rightarrow OP > 2$ .<sup>9</sup> Use the expressions for  $R^L$  and  $R^{OD}$  to write  $\frac{R^T - R^D}{R^{OD} - R^D} = \frac{(1 - rr)EFP + rrR^D}{(1 - rr)(OP)EFP + rrR^D}$ . In practice, the reserve ratio is a small fraction of deposits when the interbank interest rate is non-zero. Therefore, velocity is approximately constant at  $V[\frac{1}{OP}]$  when  $R^D > 0$  and absolutely constant at  $V[\frac{1}{OP}]$  when  $R^D = 0$ . Finally, express a household's beginning-of-period real deposit demand as

$$\frac{D^*}{P} = \frac{c}{V} \quad (5)$$

where  $V \equiv V[\frac{1}{OP}]$ .

The remaining banking sector equilibrium condition involves the banking system balance sheet constraint:  $L = (1 - rr)D$ . Use (3) and (5) to substitute for  $L$  and  $D$  in the banking balance sheet constraint to write

$$c = \frac{VF}{1 - rr} \left( \frac{B}{P} + kqK \right)^\alpha (\gamma dm)^{1 - \alpha} \quad (6)$$

This broad liquidity provision condition is written with  $c$  on the left hand side because it will be used to substitute for  $c$  in the household expected utility maximization problem

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<sup>9</sup> An alternative specification with similar implications to the one in the text is  $R^{OD} = OP[EFP + \frac{R^D}{1 - rr}]$ .

in Section 4.<sup>10</sup>

## 4 The Household Maximization Problem

Assume that the economy is populated with a large number of infinitely-lived households each of which maximizes expected lifetime utility<sup>11</sup>

$$E_0 \sum_{t=0}^{\infty} (1 + \rho)^{-t} [\phi \log c_t + (1 - \phi) \log l_t + \theta \log(\frac{C_{t+1}}{P_t})] \quad (7)$$

subject to the intertemporal budget constraint

$$(q_t + d_t)K_t + \frac{B_t}{P_t} + \frac{C_t}{P_t} + w_t(m_t^s - m_t^d) - c_t - q_t K_{t+1} - \frac{B_{t+1}}{P_t} \left( \frac{1}{1 + R_t^B} \right) - \frac{C_{t+1}}{P_t} - T_t = 0 \quad (8)$$

a time constraint

$$l_t = 1 - m_t^s \quad (9)$$

and broad liquidity constraint (6)<sup>12</sup>

$$c_t = \frac{VF}{1 - rr} \left( \frac{B_{t+1}}{P_t} + kq_t K_{t+1} \right)^\alpha (\gamma d_t m_t^d)^{1-\alpha} \quad (10)$$

where

$\rho \equiv$  the rate of time preference

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<sup>10</sup>Note that  $m$  effort supports loans made at the beginning of a period. Effort is also required for banks to make payments and to arrange overdrafts at the end of the period. An individual household does not know in advance whether and how much  $it$  will need to overdraft. Aggregate overdrafts, however, can be predicted with great accuracy, as can aggregate payments. Additional effort needed to make payments and to produce overdrafts is assumed to be known at the beginning of the period and to be taken into account in the beginning-of-period labor market clearing conditions. Effort involved in providing payments and overdraft services is assumed to be a fixed proportion of  $m$ , so that  $m$  alone can be interpreted as taking account of payment and overdraft effort implicitly.

<sup>11</sup>The money-in-the-utility-function specification of currency demand is chosen for its simplicity, so as not to obscure the focus on broad liquidity in the model. Moreover, the model is designed to analyze the implications for interest rate policy of factors affecting broad liquidity. And the demand for narrow money (currency and bank reserves) is automatically accommodated at the current interest rate target so that interest rate policy need not be modified to take account of narrow monetary conditions. Therefore, the details of the demand for currency are second order for the issues at hand.

<sup>12</sup>Broad liquidity constraint (10) plays a role here analogous to the role played by the transactions constraint in shopping time models of narrow money demand. See Lucas (2000) and McCallum and Goodfriend (1987).

$c_t \equiv$  consumption in period  $t$

$l_t \equiv$  leisure in period  $t$

$m_t^s \equiv$  loan production effort supplied by a household in period  $t$

$m_t^d \equiv$  loan production effort demanded by a household in period  $t$

$w_t \equiv$  the consumption wage in period  $t$

$C_t \equiv$  currency carried from period  $t-1$  into period  $t$  yielding utility transactions services in period  $t-1$  but no collateral services

$K_t \equiv$  capital goods carried from period  $t-1$  into period  $t$  yielding a risky consumption-good dividend in period  $t$  and collateral services in period  $t-1$  loan production

$d_t \equiv$  units of non-storable consumption goods yielded in period  $t$  per unit of  $K_{t-1}$  carried from  $t-1$  into  $t$

$B_t \equiv$  one-period nominally denominated government bonds carried from  $t-1$  into  $t$  yielding a riskless nominal return in period  $t$  and collateral services in period  $t-1$  loan production

$R_t^B \equiv$  the net riskless nominal return on a nominal government bond carried from  $t$  into  $t+1$

$P_t \equiv$  the price level in period  $t$

$q_t \equiv$  the consumption price of capital in period  $t$

$T_t \equiv$  lump-sum taxes or transfers that balance the consolidated central bank and government budget in period  $t$ .<sup>13</sup>

The household's constrained maximization problem may be solved by forming the Lagrangian with (8), using (9) and (10) to eliminate  $l_t$  and  $c_t$  respectively, and choosing sequences for  $m_t^s$ ,  $m_t^d$ ,  $C_t$ ,  $K_t$ , and  $B_t$  and the Lagrangian multiplier  $\lambda_t$ , given initial conditions  $C_0$ ,  $K_0$ , and  $B_0$ , and the wage  $w_t$ , the price level  $P_t$ , the price of capital  $q_t$ , the bond rate  $R_t^B$ , the consumption dividend  $d_t$ , and taxes/transfers  $T_t$  for  $t=0, 1, 2, 3, \dots$

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<sup>13</sup>The central bank and the government manage the stocks of high-powered money and government bonds to maintain price stability as discussed below. There is no public production of goods or services in the model.



The first order condition for the choice of  $m_t^s$  is

$$\frac{(1-\phi)}{1-m_t^s} = \lambda_t w_t \quad (11)$$

The first order condition for the choice of  $m_t^d$  is

$$\frac{(1-\alpha)c_t}{m_t^d} \left( \frac{\phi}{c_t} - \lambda_t \right) = \lambda_t w_t \quad (12)$$

The first order condition for the choice of  $C_{t+1}$  is

$$\left( \frac{\theta}{C_{t+1}/P_t} \right) \frac{1}{P_t} - \frac{\lambda_t}{P_t} + \frac{1}{1+\rho} E_t \left[ \frac{\lambda_{t+1}}{P_{t+1}} \right] = 0 \quad (13)$$

The first order condition for the choice of  $K_{t+1}$  is

$$\frac{\phi}{c_t} \Omega_t k q_t - \lambda_t q_t (1 + \Omega_t k) + E_t \left[ \frac{\lambda_{t+1} (q_{t+1} + d_{t+1})}{1 + \rho} \right] = 0 \quad (14)$$

where  $\Omega_t \equiv \frac{\alpha c_t}{\frac{B_{t+1}}{P_t} + k q_t K_{t+1}} \equiv$  the partial derivative of the right hand side of broad liquidity constraint (10) in period  $t$  with respect to total household weighted collateral.

Finally, the first order condition for the choice of  $B_{t+1}$  is

$$\frac{\phi}{c_t} \frac{\Omega_t}{P_t} - \frac{\lambda_t}{P_t} \left( \frac{1}{1 + R_t^B} + \Omega_t \right) + \frac{1}{1 + \rho} E_t \left[ \frac{\lambda_{t+1}}{P_{t+1}} \right] = 0 \quad (15)$$

In addition to the five first order conditions, it is useful to derive a first order condition for the choice of an *imaginary* one-period nominal bond  $B_t^T$  carried from period  $t-1$  into  $t$  yielding a riskless nominal return in period  $t$  but no collateral services in loan production. Such an imaginary bond is of interest because its explicit nominal yield would be the required *total* net nominal yield  $R_t^T$  consistent with macroeconomic equilibrium. To derive the first order condition for such a nominal bond, add the terms  $\frac{B_t^T}{P_t} - \frac{B_{t+1}^T}{P_t} \left( \frac{1}{1 + R_t^T} \right)$  to intertemporal budget constraint (8) and find the first order condition for the choice of  $B_{t+1}^T$

$$1 = \frac{1 + R_t^T}{1 + \rho} E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \right] \quad (16)$$

## 5 Market Clearing Conditions and Preliminary Solutions for the External Finance Premium

The model is closed with five market clearing conditions and an objective for monetary and fiscal policy. The market clearing conditions are these:

Labor market clearing:  $m_t^s = m_t^d$

Goods market clearing:  $c_t = d_t K$

Capital market clearing:  $K_t^d = K$

Bond market clearing:  $B_t^d = B_t$

High-powered money market clearing:  $C_t + rrD_t = H_t$

Labor market clearing requires that hours of loan production effort supplied by the average household  $m_t^s$  equal hours of loan production effort demanded  $m_t^d$ . Dividends from capital are the only source of consumption goods. Goods are not storable and the capital stock is assumed fixed and normalized at  $K = 1$ , so goods market clearing requires  $c_t = d_t$ . The capital dividend  $d_t$  reflects stochastic exogenous productivity growth. Bond market clearing requires households to hold the stock of bonds  $B_t$  supplied by the consolidated monetary and fiscal authorities. High-powered money market clearing requires that the stock of high-powered money  $H_t$  is held as either currency  $C_t$  or bank reserves  $rrD_t$ .

The central bank, with the cooperation of the fiscal authorities, is assumed to use open market operations in  $H_t$  and  $B_t$  in support of interest rate policy to perfectly stabilize the path of the price level at  $P_0 = P$  with an inflation target of  $\Pi = \frac{P_{t+1}}{P_t} - 1$ .

Under these conditions the model determines the consumption price of capital  $q_t$ , effort in loan production  $m_t$ , the consumption wage  $w_t$ , net nominal interest rates  $R_t^B$ ,  $R_t^D$ ,  $R_t^L$ , and  $R_t^T$ , consumption  $c_t$ , currency  $C_t$ , deposits  $D_t$ , high-powered money  $H_t$ , and bonds  $B_t$ . From these primary variables the model determines the consumption return on capital  $r^K$ , the external finance premium  $EFP$ , and the composition of asset returns between the implicit broad liquidity services yield and the explicit pecuniary yield.

In what follows the equilibrium solution for the model is approximated by using the first order conditions of a “representative” household that holds average per capita wealth. While the model does not admit a representative household in the technical sense, the approximate solution studied here should embody many if not most of the features of the solution that takes full account of household heterogeneity.<sup>14</sup>

The remainder of this section presents some preliminary relationships that are central to understanding the consequences of broad liquidity considerations for interest rate policy. The first step in solving the model is to express  $\lambda_t$  in terms of  $m_t$  and  $d_t$ , using

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<sup>14</sup>Technically, an economy that admits a representative agent is one where model outcomes satisfy exactly the optimality conditions of a single agent. See Krusell and Smith (1998).

(11) and  $m_t^s = m_t^d$  to eliminate  $w_t$  in (12), to obtain

$$\lambda_t = [1 - \frac{(1-\phi)}{(1-\alpha)\phi} \frac{m_t}{(1-m_t)}] \frac{\phi}{d_t} \quad (17)$$

Expression (17) embodies a central insight. The bracket term in (17) is positive and less than unity if  $m_t > 0$ . This means that an equilibrium with broad liquidity provision,  $m_t > 0$ , is one in which  $\frac{\phi}{c_t} > \lambda_t$ . In such an equilibrium a household stops short of raising consumption to the usual point where the marginal utility of consumption (MUC) is brought into equality with the marginal utility of income (MUI). The reason is that according to (5) optimal real deposit demand rises with consumption and acts like a marginal tax on consumption that drives a wedge between MUC and MUI. The tax arises because in equilibrium deposits must be financed by bank loans that are costly to produce. The implicit tax wedge is reflected in the bracket term in (17). Inspecting the term inside the brackets, the implicit tax rate rises with  $m_t$  because of diminishing marginal product of effort in loan production and diminishing marginal utility of leisure.

Now substitute for  $\lambda_t$  in (11) using (17) to express the real wage in terms of  $m_t$  and  $d_t$

$$w_t = \frac{d_t/\phi}{\frac{1-m_t}{(1-\phi)} - \frac{m_t}{\phi(1-\alpha)}} \quad (18)$$

In expression (18), higher  $d_t$  lowers MUC and therefore requires a higher real wage to clear the labor market given  $m_t$ . Higher  $m_t$  requires a higher real wage to clear the labor market given the MUC because of diminishing marginal utility of leisure.

Use (18) to substitute for the real wage  $w \equiv W/P$  in (4) to express the  $EFP_t$  in terms of  $m_t$ ,  $d_t$ , and  $q_t$

$$EFP_t = \frac{1}{[\frac{\phi(1-m_t)}{(1-\phi)} - \frac{m_t}{(1-\alpha)}]} \frac{(\gamma d_t m_t)^\alpha}{(1-\alpha)F(\frac{B_{t+1}}{P_t} + k q_t K)^\alpha \gamma} \quad (19)$$

In (19) we see that the  $EFP$  rises with  $m_t$  because of the higher real wage and diminishing marginal product of effort in loan production. Note that a higher consumption price of capital  $q_t$  lowers the  $EFP_t$  because it raises the value of collateral in loan production.

Expressions (20) and (21) below are two other preliminary solutions for the  $EFP_t$ . The first is obtained from (4) and (10) with  $c_t = d_t$

$$EFP_t = \frac{V}{(1-rr)(1-\alpha)} \frac{w_t m_t}{d_t} \quad (20)$$

The second uses (18) to substitute for  $w_t$  in (20)

$$EFP_t = \frac{V}{(1 - rr)} \frac{1}{\left[\frac{\phi(1-\alpha)}{(1-\phi)} \frac{(1-m_t)}{m_t} - 1\right]} \quad (21)$$

Using (17) we can equate

$$\frac{w_t m_t}{d_t} = \frac{1}{\left[\frac{\phi}{(1-\phi)} \frac{(1-m_t)}{m_t} - \frac{1}{(1-\alpha)}\right]} = (1 - \alpha) \left(\frac{\phi}{\lambda_t d_t} - 1\right) \quad (22)$$

Expression (20) gives the  $EFP_t$  in terms of the ratio of the value of effort in loan production relative to consumption, a useful measure of the cost of broad liquidity. And (21) expresses that ratio and  $EFP_t$  in terms of  $m_t$  alone. Finally, (22) gives three representations of the cost of broad liquidity. Note that the size of the right-most term is directly related to the excess of the MUC over the MUI, reflecting the tax wedge discussed with respect to (17) above.

## 6 Currency Demand and the Price Level

The purpose of this section is to derive the household's demand for currency and to use that demand function to derive a relationship between high-powered money and the price level. To derive the real demand for currency combine the first order conditions for currency and government bonds, (13) and (15) respectively, to eliminate the " $E_t$ " terms and obtain

$$\frac{\theta}{\frac{C_{t+1}}{P_t}} = \frac{\lambda_t R_t^B}{1 + R_t^B} + \Omega_t \left(\frac{\phi}{d_t} - \lambda_t\right) \quad (23)$$

where  $\Omega_t$  is defined in Section 4. According to (23) currency is chosen to equate the marginal utility of real balances to the opportunity cost. The first opportunity cost term reflects the usual interest cost foregone converted to utility terms by the  $\lambda_t$  multiplier. The second opportunity cost term is new. It represents the marginal implicit services yield provided by bonds as collateral in loan production and reflects the fact that currency yields *no* collateral services in loan production.

It is useful to derive the demand for currency in terms of  $R_t^T$  instead of  $R_t^B$ . To do so, use the first order conditions for currency and the imaginary bond that yields no collateral services, (13) and (16) respectively, to obtain

$$\frac{\theta}{\frac{C_{t+1}}{P_t}} = \frac{\lambda_t R_t^T}{1 + R_t^T} \quad (24)$$

There is only an interest opportunity cost term in (24) because  $R_t^T$  is the total nominal return on a bond yielding no collateral services in loan production.

The relationship between the price level and high-powered money is derived as follows. Use (5) and goods market clearing to write

$$P_t = \frac{V}{d_t} D_{t+1} \quad (25)$$

And use the high-powered money market clearing condition to substitute for  $D_{t+1}$  in (25) to obtain

$$P_t = \frac{V}{d_t} \left[ \frac{H_{t+1} - C_{t+1}}{rr} \right] \quad (26)$$

Next, substitute for  $C_{t+1}$  and  $\lambda_t$  from (24) and (17) respectively to obtain

$$P_t = \left[ 1 / \left[ \frac{rr}{V} + \frac{\theta}{\left( \phi - \frac{(1-\phi)m_t}{(1-\alpha)(1-m_t)} \right) \left( \frac{1}{R_t^T} + 1 \right)} \right] \right] \frac{H_{t+1}}{d_t} \quad (27)$$

The price level is proportionate to the ratio of high-powered money to goods income in the model, with the coefficient of proportionality dependent as usual on deposit velocity and the reserve ratio, as well as on a nominal interest rate. What is new is the potential influence of loan production effort  $m_t$  on the price level operating through  $\lambda_t$ , and through the effect of  $\lambda_t$  on  $R_t^T$  according to the first order condition (16) that determines  $R_t^T$ .<sup>15</sup>

We assumed in Section 5 that the central bank, with the cooperation of the fiscal authorities, uses open market operations to perfectly stabilize the path for the price level. By inverting (27) we can obtain the policy behavior for  $H_{t+1}$  that does so. Note that the effect of open market operations on the stock of bonds also influences the path for high-powered money that is consistent with price stability. The bond stock  $B_{t+1}$  does not appear directly in (27), but it influences  $m_t$  and  $R_t^T$  indirectly through the weighted value of collateral  $\frac{B_{t+1}}{P_t} + kq_t K$ .

## 7 Interest Rates, Rate of Return Spreads, and Implicit Broad Liquidity Service Yields

For the purpose of understanding the factors determining interest rates, rate of return spreads, and implicit liquidity service yields it is convenient to express the various rates of return in the model in relation to the riskless total nominal interest rate  $R_t^T$ . Assuming that the central bank targets inflation exactly,  $R_t^T$  is determined implicitly by imposing  $\frac{P_{t+1}}{P_t} = 1 + \Pi$  in first order condition (16) for the household choice of the imaginary bond yielding no collateral services

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<sup>15</sup>Using currency demand function (23) instead of (24), an alternative relationship between the price level and high-powered money can be derived in terms of the explicit pecuniary bond rate  $R_t^B$ .

$$1 = \frac{1 + R_t^T}{(1 + \rho)(1 + \Pi)} E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\right] \quad (28)$$

The determinants of the  $R_t^T$  nominal interest rate look familiar. However, note that the production and use of broad liquidity services in the model influences the benchmark nominal interest rate  $R_t^T$  through  $\lambda_t$  and  $E_t[\lambda_{t+1}]$  according to (17).

To determine the government bond rate  $R_t^B$  implicitly, use the first order conditions for the government bond and the benchmark bond, (15) and (16) respectively, together with  $c_t = d_t$  to obtain

$$\frac{1}{1 + R_t^B} - \frac{1}{1 + R_t^T} = \Omega_t\left(\frac{\phi}{\lambda_t d_t} - 1\right) \quad (29)$$

Multiply (29) by  $(1 + R_t^B)(1 + R_t^T)$  and approximate the implicit broad liquidity services yield on government bonds as

$$LSY_t^B = R_t^T - R_t^B = \Omega_t\left(\frac{\phi}{\lambda_t d_t} - 1\right) \quad (30)$$

Expression (30) identifies the spread between the total nominal yield in asset markets  $R_t^T$  and the explicit pecuniary interest on government bonds  $R_t^B$  as the implicit liquidity services yield  $\Omega_t(\frac{\phi}{\lambda_t d_t} - 1)$ . This verifies that the opportunity cost term discussed in connection with the demand for currency (23) is approximately the implicit broad liquidity services yield on government bonds.<sup>16</sup>

Define  $r_t^K \equiv \frac{q_{t+1} + d_{t+1}}{q_t} - 1 \equiv$  the net explicit real pecuniary return on capital carried from period  $t$  to  $t+1$ . To determine the net expected explicit real return on capital  $E_t r_t^K$  use the first order conditions for capital and the benchmark bond, (14) and (16) respectively, together with  $c_t = d_t$ , and  $\frac{P_{t+1}}{P_t} = 1 + \Pi$  to write

$$E_t r_t^K = \frac{1 + R_t^T}{1 + \Pi} \left\{ 1 - k \Omega_t\left(\frac{\phi}{\lambda_t d_t} - 1\right) - \frac{1}{(1 + \rho)} Cov_t\left[\frac{\lambda_{t+1}}{\lambda_t}, r_t^K\right] \right\} - 1 \quad (31)$$

Multiply (31) by  $(1 + \Pi)$  and approximate the liquidity services yield on capital as

$$LSY_t^K = (R_t^T - \Pi) - E_t r_t^K - Cov_t\left[\frac{\lambda_{t+1}}{\lambda_t}, r_t^K\right] = k \Omega_t\left(\frac{\phi}{\lambda_t d_t} - 1\right) \quad (32)$$

Comparing (30) and (32) we see that  $LSY_t^K = k LSY_t^B$ . The liquidity services yield on capital is a fraction  $k$  of that on bonds because  $k$  reflects the lower productivity in

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<sup>16</sup>This result is related to Huggett's (1993) finding that the equilibrium risk-free interest rate is below the rate of time preference when agents experience uninsurable idiosyncratic endowment shocks and smooth consumption by holding the risk-free asset. Using the terminology developed here, we would say that the positive spread between the rate of time preference and the risk-free rate in Huggett's model represents the implicit broad liquidity services yield on the risk-free asset.

(3) of capital relative to bonds as collateral in loan production. Furthermore, from (20) and (22) we see the close relationship between the liquidity service yields and the external finance premium:  $LSY_t^K = kLSY_t^B = k\Omega_t[\frac{1-rr}{V}]EFP_t$ . In particular, note that the liquidity service yields are positive if and only if there is an external finance premium.

Rewrite (32) as  $E_tr_t^K = (R_t^T - \Pi) - LSY_t^K - Cov_t[\frac{\lambda_{t+1}}{\lambda_t}, r_t^K]$  to see that the equilibrium expected explicit real pecuniary return to capital  $E_tr_t^K$  depends on three factors: (1) the inflation-adjusted riskless nominal interest rate  $R_t^T - \Pi$  on a bond yielding no liquidity services in loan production, (2) the implicit broad liquidity services yield on capital  $k\Omega_t(\frac{\phi}{\lambda_t d_t} - 1)$ , and (3) and a risk compensation term  $Cov_t[\frac{\lambda_{t+1}}{\lambda_t}, r_t^K]$ . Note that  $E_tr_t^K$  can be either higher or lower than  $R_t^T - \Pi$ . The  $LSY_t^K$  pushes  $E_tr_t^K$  down relative to  $R_t^T - \Pi$  given  $Cov_t[\frac{\lambda_{t+1}}{\lambda_t}, r_t^K]$ .<sup>17</sup> However, the risk term  $Cov_t[\frac{\lambda_{t+1}}{\lambda_t}, r_t^K]$  pushes the expected explicit real pecuniary yield on capital up, since returns surprises are negatively correlated with MUI surprises and households must be compensated with a higher expected explicit return for bearing risk.<sup>18</sup>

Finally, there are two nominal interest rates in the model that need to be determined, the nominal deposit rate  $R_t^D$  and the nominal interbank interest rate  $R_t^{IB}$ . The deposit rate is determined by the equality of marginal cost and marginal revenue in loan production  $R_t^L = EFP_t + \frac{R_t^D}{1-rr}$  together with the no arbitrage condition  $R_t^L = R_t^T$  between the loan market and the asset market

$$R_t^D = (1 - rr)(R_t^T - EFP_t) \quad (33)$$

The interbank interest rate is determined by the equalization of the cost of alternative sources of loanable funds and by the fact that banks do not hold reserves against interbank balances so that  $R_t^{IB} = \frac{R_t^D}{1-rr}$  and

$$R_t^{IB} = R_t^T - EFP_t \quad (34)$$

where the  $EFP_t$  is determined according to (19), (20), or (21). The interbank rate plays a key role as the interest rate policy instrument in Sections 8 and 9 below.

<sup>17</sup>This result is related to Ayagari's (1994) finding that agents are willing to accumulate capital in the steady state past the point that its net marginal product equals the rate of time preference, when capital provides self-insurance against idiosyncratic income risk. Using the terminology developed here, we would say that the positive spread between the rate of time preference and the marginal product of capital in Ayagari's model represents the implicit broad liquidity services yield on capital.

<sup>18</sup>The specification of log utility assumed here governs the required compensation for risk.

## 8 The “Neutral” Interbank Rate in Balanced Growth with Aggregate Perfect Foresight

The full equilibrium of the model economy is presented and characterized in this section for the case of balanced growth with aggregate perfect foresight. The model is used to show how the “neutral” interbank interest rate depends on factors related to the production and use of broad liquidity.

### 8.1 The Balanced Growth Equilibrium

The balanced growth equilibrium with aggregate perfect foresight is found by assuming that  $\frac{d_{t+1}}{d_t} = 1 + g$ ,  $\frac{P_{t+1}}{P_t} = 1 + \Pi$ , and  $\frac{B_{t+1}}{P_t}/d_t K = \hat{B}$ , where  $g \equiv$  productivity growth,  $\Pi \equiv$  the inflation rate, and  $\hat{B} \equiv$  a constant ratio of real government bonds to output. Define the constant  $Q$  such that:  $q_t = \frac{d_t}{\rho} Q$ . In what follows we seek the constant values of loan production effort  $m$  and the stationary transform of the consumption price of capital  $Q$  that satisfy the equilibrium conditions of the model.

The balanced growth conditions imply:  $\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{1+g}$ ,  $\frac{q_{t+1}}{q_t} = 1 + g$ , and  $\frac{d_{t+1}}{q_t} = \frac{\rho}{Q}(1 + g) \implies \frac{\lambda_{t+1}}{\lambda_t}(\frac{q_{t+1} + d_{t+1}}{q_t}) = 1 + \frac{\rho}{Q}$ .

Starting with the first order condition for capital (14), use the above conditions implied by balanced growth, equation (22), the definition of  $Q$ , the definition of  $\Omega_t$ , and  $c_t = d_t$  to derive an equilibrium condition that reflects capital, goods, and labor market clearing. The “CL” condition is

$$1 - \frac{\alpha k}{K(\hat{B} + \frac{k}{\rho} Q)} \left[ \frac{1}{\frac{(1-\alpha)\phi}{1-\phi}(\frac{1-m}{m}) - 1} \right] = \frac{1}{1+\rho} \left( 1 + \frac{\rho}{Q} \right) \quad (35)$$

CL condition (35) is the first of two equilibrium conditions that determine  $m$  and  $Q$  (and  $q_t$  indirectly) on a balanced growth path. The capital price transform  $Q$  affects the CL condition through two channels: (1) higher  $Q$  lowers the right hand side of (35) by lowering the explicit pecuniary return and (2) higher  $Q$  raises the left hand side of (35) by lowering the implicit broad liquidity services yield on capital because of diminishing marginal productivity of collateral in loan production. Loan production effort  $m$  affects the CL condition through its effect on the liquidity services yield: higher  $m$  lowers the left hand side of (35) by raising the marginal product of collateral in loan production, thereby raising the implicit broad liquidity services yield on capital as collateral.

Note the following features of (35). First, if  $m = 0$ , then (35) reduces to  $Q = 1$ , which implies that  $q_t = \frac{d_t}{\rho}$  and  $r^K = R^T - \Pi = (1 + g)(1 + \rho) - 1$ . Second, if  $m > 0$ , then  $Q > 1$ ,  $q_t > \frac{d_t}{\rho}$ , and  $r^K = (1 + g)(1 + \frac{\rho}{Q}) - 1$ . Third, if  $m > 0$ , then  $LSY^K = (R^T - \Pi) - r^K = \rho(1 + g)(1 - \frac{1}{Q})$ .



The second equilibrium condition is derived by imposing  $c_t = d_t$  and  $\frac{B_{t+1}}{P_t}/d_t K = \hat{B}$  on broad liquidity constraint (10) and using the definition of  $Q$ . The “BL” condition is

$$1 = \frac{VF}{1 - rr} (K(\hat{B} + \frac{k}{\rho}Q))^\alpha (\gamma m)^{1-\alpha} \quad (36)$$

BL condition (36) is a loan market clearing condition that reflects the banking system balance sheet constraint and embodies factors governing the demand for deposits and the production of loans. The capital price transform  $Q$  and loan production effort  $m$  affect the BL condition through their positive effect on loan production.

CL condition (35) and BL condition (36) simultaneously determine constant values of  $Q$  and  $m$  along the balanced growth path. The equilibrium is most easily characterized by imagining the two equilibrium conditions drawn in a space with  $Q$  on the vertical axis and  $m$  on the horizontal axis. The CL locus slopes upward and is convex from below; it intersects the vertical axis at  $Q = 1$  and asymptotes to a vertical line at  $m = \frac{1}{1 + \frac{(1-\phi)}{(1-\alpha)\phi}}$ . The BL locus slopes downward and is also convex from below; it intersects the horizontal axis at  $m = ([\frac{1-rr}{VF(K\hat{B})^\alpha}]^{\frac{1}{1-\alpha}})/\gamma$  and asymptotes to the vertical axis. Hence, an equilibrium exists where the two loci intersect at  $Q > 1$  and  $0 < m < 1$ . From  $q_t = \frac{d_t}{\rho}Q$  we see that the consumption price of capital  $q_t$  grows at the productivity growth rate  $g$  on a path that is permanently higher than if collateral services were not valued.<sup>19</sup> Moreover, we see that implicit broad liquidity service yields are positive:  $LSY^K = kLSY^B = \rho(1+g)(1 - \frac{1}{Q}) > 0$ . From (19) or (21) we see that the external finance premium  $EFP$  is positive as well.

## 8.2 The “Neutral” Interbank Interest Rate

The Federal Reserve and other central banks typically implement monetary policy with a nominal interbank interest rate policy instrument. To help guide interest rate policy in practice, it is useful to have an idea of the “neutral” level of the nominal interbank rate that is consistent with a balanced economic expansion and the price level stabilized on a path consistent with the central bank’s inflation target. We interpret the level of the interbank rate  $R^{IB}$  in balanced growth as the “neutral” interbank rate and investigate its determinants below.

From (34) we know that  $R^{IB} = R^T - EFP$ . Imposing the relevant balanced growth and inflation targeting conditions in (16), we approximate  $R^T = \rho + g + \Pi$ . Thus, we see that the total nominal interest rate  $R^T$  in balanced growth is determined exclusively by

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<sup>19</sup>In the standard one-sector growth model with investment, the consumption price of capital would be unity on a balanced growth path and the capital stock would be permanently above what it would be if broad liquidity (and collateral services) were not valued.

the standard factors: productivity growth, time preference, and the inflation rate. The neutral interbank rate inherits these standard determinants.

However, the neutral interbank rate depends in balanced growth on factors affecting the production and use of broad liquidity through the external finance premium. It is beyond the scope of this paper to explore the magnitude of broad liquidity effects on the neutral interbank rate. Instead, we describe below the nature of the influence of broad liquidity considerations by showing how the following structural shifts influence the external finance premium:

(1) A reduction in  $F$  or  $\gamma$  that reflects lower productivity of collateral and effort in loan production shifts the BL locus up in  $Q, m$  space and raises equilibrium  $Q$  and  $m$ . The  $EFP$  rises, too, according to (21) and pushes  $R^{IB}$  down since  $R^T$  is unaffected.

(2) A reduction in velocity  $V$  or a decrease in reserve ratio  $rr$  shifts the BL locus up and raises  $Q$  and  $m$ . According to (21) the effect on  $EFP$  is ambiguous in this case, as is the effect on the neutral interbank rate.

(3) An increase in uninsurable idiosyncratic liquidity risk, perhaps associated with a higher growth rate  $g$  and more intense job creation and destruction, affects both the production and use of broad liquidity. On the production side,  $F$  and  $\gamma$  fall as more collateral and effort are required to manage and monitor loans. And on the use side, velocity falls as households increase their deposits. In addition, the total required nominal interest rate rises with productivity growth. Multiple impacts on the demand and supply of broad liquidity such as these have the potential to exert particularly large effects on the external finance premium and the neutral interbank rate.

(4) An increase in the outstanding stock of government bonds  $\hat{B}$  relative to output raises the weighted value of collateral in the economy and thereby shifts the BL and the CL locus down. Since BL shifts down by more than CL, both  $Q$  and  $m$  fall, as does the  $EFP$  according to (21). Since  $R^T$  is unaffected, the neutral interbank rate  $R^{IB}$  rises. Other things the same, the neutral interbank rate would tend to be positively correlated with the stock of government bonds outstanding.

[Note: In this model with non-distorting lump sum taxes, the increased stock of bonds has two countervailing effects on welfare: (1) it increases household utility by reducing equilibrium effort  $m$  in loan production, and (2) it reduces household utility according to (17) and (24) by raising  $\lambda_t$  on the balanced growth path and reducing real currency balances held by households. This negative effect actually undoes a benefit of  $m > 0$  that pushes  $\lambda_t$  below  $\frac{\phi}{d_t}$ , and helps to offset the underutilization of currency due to a positive nominal interest rate.<sup>20</sup>]

(5) An increase in trend productivity growth  $g$  or in the inflation target  $\Pi$  raises  $R^T$  but has no effect on  $Q$ ,  $m$ ,  $EFP$ , or the  $LSYs$ .

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<sup>20</sup>See Ayagari and McGrattan (1998) for a study of the optimum quantity of government debt in an incomplete markets model in which bonds provide self-insurance.

(6) An increase in the level of the path of productivity,  $d_t$ , holding  $g$  constant, raises the consumption price of capital according to  $q_t = \frac{Q}{\rho}d_t$  but has no effect on  $Q$ ,  $m$ ,  $EFP$ , or the  $LSY$ s.

(7) Finally, the non-linear nature of the BL and CL loci in  $Q, m$  space means that the sensitivity of  $Q$ ,  $m$ ,  $EFP$ , and the  $LSY$ s to structural shifts such as those described above could vary significantly depending on the relative positions of the loci which, among other things, depend on the stock of government bonds outstanding.

Bottom line: A model of monetary policy that ignores broad liquidity considerations is incomplete and potentially highly misleading as a guide to factors affecting the neutral interbank interest rate in balanced growth with inflation stabilized at the central bank's target.

## 9 Interest Rate Policy Actions and Broad Liquidity

Broad liquidity considerations matter not only for the *neutral* interbank rate but also for interbank interest rate policy *actions* that must be taken to stabilize inflation at the central bank's target in response to shocks that hit the economy. This section shows how and why interest rate policy actions must be modified to take account of broad liquidity. We study a variety of shocks to the macroeconomy below using *short-run* equilibrium conditions that correspond to the CL and BL conditions derived and utilized to study balanced growth in Section 8.

The central bank is presumed to follow a fully credible interest rate rule that perfectly stabilizes the path of the price level. It is assumed to understand the structure of the economy, including the production and use of broad liquidity, and to observe the shocks contemporaneously. The interest rate rule requires the central bank to set its interbank rate policy instrument  $R_t^{IB}$  each period, conditional on observing the shock(s), so that the loan rate  $R_t^L$  tracks exactly the total nominal interest rate  $R_t^T$  necessary to keep inflation on target. The analysis proceeds in four steps.

First, determine the consequences of the shock(s) for  $q_t$  and  $m_t$  using two short-run equilibrium conditions. The short-run "SRCL" condition is derived from the first order condition for capital (14), broad liquidity constraint (10), the determinants of  $\lambda_t$  (17), the definition of  $\Omega_t$ ,  $c_t = d_t$ , and  $K_{t+1} = K$

$$1 - \frac{1 - \phi}{(1 - \alpha)\phi} \left( \frac{m_t}{1 - m_t} \right) \left[ 1 + \frac{k\alpha d_t}{\frac{B_{t+1}}{P_t} + kq_t K} \right] = \frac{d_t}{\phi q_t} E_t \frac{[\lambda_{t+1}(q_{t+1} + d_{t+1})]}{1 + \rho} \quad (37)$$

The short-run "SRBL" condition is derived from broad liquidity constraint (10) with  $c_t = d_t$ , and  $K_{t+1} = K$

$$d_t^\alpha = \frac{VF}{1 - rr} \left( \frac{B_{t+1}}{P_t} + kq_t K \right)^\alpha (\gamma m_t)^{1-\alpha} \quad (38)$$

The SRCL and SRBL conditions, (37) and (38), simultaneously determine  $q_t$  and  $m_t$  taking  $d_t$  and  $E_t[\lambda_{t+1}(q_{t+1}+d_{t+1})]$  as exogenous. The  $\frac{B_{t+1}}{P_t}$  term is largely predetermined since the price level is presumed to be stabilized along a predetermined path and the nominal stock of government bonds mostly results from past government borrowing. In what follows we ignore the change in  $B_{t+1}$  associated with open market operations in  $H_{t+1}$  required to offset the effect of period  $t$  shocks on the price level according to (27). The channels of influence of  $q_t$  and  $m_t$  are identical to those discussed with respect to the CL and BL conditions in balanced growth. The short-run conditions have the same shapes diagrammatically as their balanced-growth counterparts, except that we can think of the two loci in  $q, m$  instead of in  $Q, m$  space. SRCL intersects the vertical axis at  $q_t = \frac{d_t}{\phi} E_t[\lambda_{t+1}(q_{t+1} + d_{t+1})]$  and SRBL intersects the horizontal axis at  $m_t = ([\frac{d_t^\alpha(1-rr)}{VF(\frac{B_{t+1}}{P_t} + kq_t K)^\alpha}]^{\frac{1}{1-\alpha}})/\gamma$ .

Second, determine the consequences for  $\lambda_t$  from (17), and for  $R_t^T$  from (28).

Third, determine the consequences for  $EF P_t$  from (19), (20), or (21).

Fourth, determine the consequences for  $R_t^{IB}$  from (34) using the results for  $R_t^T$  and  $EF P_t$ .

We analyze three types of shocks to the macroeconomy below: (1) an expectations shock, (2) a temporary productivity shock, and (3) temporary direct shocks to the broad liquidity sector. We use the model to derive the policy action that a central bank must take with its interbank rate instrument  $R_t^{IB}$  in response to each type of shock to keep the price level on its targeted path.

## 9.1 An Expectations Shock

Consider a negative shock to the expected price of capital  $E_t q_{t+1}$ , assuming no change in  $E_t \lambda_{t+1}$ ,  $E_t d_{t+1}$ , or in the conditional covariances. The pessimism about the future is transmitted to the present through the expectations term in the SRCL condition, which shifts the SRCL locus down in  $q_t, m_t$  space. Since the SRBL locus does not shift, the shock causes  $q_t$  to fall and  $m_t$  to rise. According to (17), higher  $m_t$  reduces  $\lambda_t$ . From (28) we see that lower  $\lambda_t$  reduces  $R_t^T$  as well. And from (19) we see that lower  $q_t$  and higher  $m_t$  raise the  $EF P_t$ . Finally, from (34) we see that the central bank must lower its interbank rate policy instrument  $R_t^{IB}$  to maintain its inflation target because  $R_t^T$  falls and the  $EF P_t$  rises.

We can interpret the above effects as follows. The negative shock to the expected price of capital reduces the expected explicit pecuniary return, and the current price of capital falls to restore the total risk-adjusted return to capital. However, the lower current price of capital reduces aggregate collateral in the economy. Reduced collateral lowers the productivity of effort in loan production and requires an increase in effort to clear the loan market. The lower price of capital and greater effort in loan production

raise the liquidity services yield on capital, and help to bring the total return to capital back up the required total return. In this case broad liquidity considerations actually *attenuate* the fall in the current price of capital.

The total nominal interest rate  $R_t^T$  falls because increased effort in loan production raises the current relative to the future implicit marginal tax on consumption discussed in connection with (17) so that the total real interest rate  $R_t^T - \Pi$  must fall to clear the goods market. This second factor also attenuates the fall in the current price of capital.

The  $EF P_t$  rises for the same reason as the rise in the  $LSY_t$ , the reduction in collateral and the increase in effort raise the marginal cost of loan production.

This example was chosen because large asset price movements occur relatively frequently in the short run, and because the example nicely highlights the implications of the production and use of broad liquidity for policy actions. In this case literally *no* change in the interbank rate is needed to keep inflation on target in the absence of effects due to the production and use of broad liquidity. Yet the analysis identifies effects on both  $R_t^T$  and  $EF P_t$  due to broad liquidity considerations as reasons why the interbank rate ought to be lowered in response to a fall in asset prices.

We see from (24) that the demand for currency rises because  $\lambda_t$  and  $R_t^T$  both fall, and from (27) we see the corresponding increase in the demand for high-powered money  $H_{t+1}$ . The central bank automatically accommodates any change in the demand for narrow money in defense of its interbank rate target, whether or not it adjusts its interbank rate target in response to the shock. Thus, the consequences for interest rate policy of narrow and broad money considerations differ sharply: Interest rate policy actions need *not* be modified to take account of factors affecting narrow money demand, but they *must* be modified to take into account factors affecting the production and use of broad money in order to stabilize inflation in response to the asset price shock.

## 9.2 A Temporary Productivity Shock

Consider a negative shock to current productivity  $d_t$  expected to be transitory. Lower productivity shifts both the SRCL and the SRBL locus down in  $q_t, m_t$  space. The SRBL locus shifts down because the lower  $d_t$  consumption scale variable in the demand for deposits reduces loan demand by more than the reduced  $d_t$  productivity index in loan production reduces loan supply. Hence,  $q_t$  must fall given  $m_t$  to bring loan supply down to loan demand in order to restore SRBL condition (38).

SRCL shifts down for two reasons. First, from the discussion following (32) we know that  $LSY_t^K = k\Omega_t[\frac{1-r}{V}]EF P_t$ . And from (21) we know that  $EF P_t$  is unchanged given  $m_t$ . But since  $\Omega_t$  falls with  $d_t$ ,  $LSY_t^K$  falls also. Second, reduced  $d_t$  raises  $MUC_t$  and  $\lambda_t$  given  $m_t$  by (17), and raises  $R_t^T$  according to (28). Hence,  $q_t$  must fall given  $m_t$  to move the  $LSY_t^K$  back up and to raise the expected explicit pecuniary return to

capital  $E_t r_t^K$  in order to restore SRCL condition (37).

SRCL shifts down by less than SRBL so that both  $m_t$  and  $q_t$  fall.<sup>21</sup> From (17) we know that  $\lambda_t$  rises unambiguously both because lower  $m_t$  temporarily reduces the implicit marginal tax rate on consumption and because lower consumption temporarily raises  $MUC_t$ . Hence,  $R_t^T$  must rise to clear the goods market as indicated in (28). And we see from (21) that the  $EF P_t$  falls unambiguously.

According to (34), since  $R_t^T$  rises and  $EF P_t$  falls the central bank must increase its interbank rate target  $R_t^{IB}$  in order to hit its inflation target. For this temporary negative productivity shock, even without broad liquidity considerations  $R_t^{IB}$  would have to rise with  $R_t^T$  for the standard reason—because  $MUC_t$  is elevated relative to  $E_t MUC_{t+1}$ . Taking broad liquidity considerations into account *amplifies* the required increase in  $R_t^{IB}$  by reducing the  $EF P_t$ , and by lowering  $m_t$ , increasing  $\lambda_t$  relative to  $E_t \lambda_{t+1}$ , and raising  $R_t^T$  even further.

### 9.3 Temporary Direct Shocks to the Broad Liquidity Sector

Consider shocks that directly create an excess demand for loans: a decrease in deposit velocity  $V$ , a decrease in the bank reserve ratio  $rr$ , or a decrease in productivity coefficients  $F$  and  $\gamma$  that govern loan supply. These shocks shift the SRBL locus up in  $q_t, m_t$  space but do not shift the SRCL locus. The shocks raise both  $q_t$  and  $m_t$  because some combination of an increase in collateral values and an increase in loan production effort is needed to restore equilibrium in the loan market. Higher  $m_t$  lowers  $\lambda_t$  according to (17) because it raises the implicit tax on consumption. And temporarily lower  $\lambda_t$  also requires a lower  $R_t^T$  to clear the goods market according to (28).

The analysis of the four shocks differs from this point on. A decrease in  $F$  or  $\gamma$  unambiguously raises the  $EF P_t$  according to (21), so that the central bank must unambiguously reduce  $R_t^{IB}$  in response to a reduction of productivity in loan supply. However, from (19) or (21) we see that the effects on the  $EF P_t$  are ambiguous for the  $V$  and  $rr$  shocks, as is the direction of the interbank rate policy response necessary to keep inflation on target. Nevertheless, the lesson is that interest rate policy cannot ignore shocks originating in the broad liquidity sector. There is no automatic accommodation. In general, interbank rate policy actions must be undertaken in response to such shocks to sustain the targeted rate of inflation.

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<sup>21</sup>To see this, let  $q_t$  in SRBL fall just enough to leave the ratio  $\frac{d_t}{\frac{B_{t+1}}{P_t} + k q_t K}$  unchanged. This decrease in  $q_t$  is the downward shift in SRBL at an unchanged  $m_t$ . Now consider the same decrease in  $q_t$  at an unchanged  $m_t$  in SRCL. The left hand side of SRCL would be unchanged, but the right hand side would rise. Hence, SRCL shifts down by less than SRBL.

## 10 Conclusion

Central banks favor interest rate policy in large part because the demand for currency and bank reserves is automatically satisfied by open market operations taken in defense of an interbank rate target. This feature of interest rate targeting is especially attractive when the demand for narrow liquidity rises suddenly and substantially in times of financial stress. Unfortunately, automatic accommodation does not extend to broad liquidity—bank deposits backed by loans. Interest rate policy must be modified to offset the effect on the economy of conditions in the market for broad liquidity. We saw that broad liquidity conditions must be taken into account in the pursuit of interest rate policy for two reasons: (1) they influence the link between the interbank rate and market rates through their effect on the external finance premium, and (2) they affect the behavior of market interest rates that the central bank must target in order to maintain macroeconomic stability. We showed how the production and use of broad liquidity influences the neutral level of the interbank rate consistent with balanced growth and stable inflation, and we showed that interbank rate policy actions must be modified in light of broad liquidity considerations to stabilize inflation in response to shocks.

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